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Effects of inclined magnetic field on Poiseuille flow of micropolar fluid in a channel bounded below by a porous bed

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---*ABSTRACT*---
The paper deals with analytical study of an incompressible micropolar fluid flow through a channel bounded below by a porous bed in the presence of an inclined magnetic field. The flow in the channel is assumed to be governed by the micropolar fluid equations and that in the porous region by Darcy's law. The governing equations are analytically solved and the expressions for velocity and microrotation are derived in closed forms. The effects of the Hartmann number, inclination angle, micropolar parameter, and porosity parameters on velocity and microrotation are studied. The results are analyzed and depicted through graphs.
KEYWORDS:- *Hartmann number, inclination angle, micropolar fluid, porous bed*

I. INTRODUCTION

The theory of micro fluids was introduced by Eringen [1]. This theory was able to describe the behavior and nature of complex fluids such as muddy fluids, magnetic fluids, slurries, liquid crystals, fluids with additives etc. The micropolar fluids [2] are a subclass of micro fluids. In micropolar fluid theory, the fluid molecules can rotate independently of the fluid stream velocity, this leads to non-symmetry in the stress tensor [3]. To describe the independent rotation of the fluid particles, a kinematic vector called microrotation is introduced in the micropolar fluid model. Owing to this, one more governing equation is included known as the conservation of angular momentum. "The attractiveness and power of the model of micropolar fluids come from the fact that it is both a significant and a simple generalization of the classical Navier-Stokes model" [4].

Flow through and/or past porous media [5] has gained practical importance, more than a century, due to a variety of applications such as underground spreading of chemical waste, drainage problems, flow through ceramic materials as bricks, production of oil from underground sources, etc. Channbasappa and Ranganna [6] studied fluid flow with variable viscosity in a parallel plate channel having one porous bounding wall. Rajasekhara and Rudraiah [7] studied Couette flow past a homogeneous porous bed.

Several researchers discussed the effects of inclined magnetic field on viscous fluid flow through different geometries [8-12]. Srinivasacharya and Hima Bindu [13] studied the magnetic effects on the flow of micropolar fluid in a rectangular duct. In two different contexts, the dynamics/influence of inclined magnetic field on micropolar Casson fluid is studied in [14, 15]. Yadav et. al. [16] discussed the magnetic effects on the three layer flow through porous medium in which the micropolar fluid is in between Newtonian fluids. As per the knowledge of the authors, the study of Poiseuille flow of a micropolar fluid in a channel with lower porous bed under the influence of inclined magnetic field is not studied so far. Hence, in this paper, flow of a micropolar fluid flow through a channel bounded below by a porous bed is analyzed. By adopting, the Beavers-Joseph (BJ) slip condition [17] at the interface of micropolar fluid and porous bed, the analytical expression for velocity and microrotation are obtained. The effects of various parameters on the velocity profiles and microrotation are shown graphically and discussed.

II. FIELD EQUATIONS

The micropolar fluid equations are

$$\frac{d\rho}{dt} + \rho\,\nabla.(\bar{q}) = 0 \tag{1}$$

$$\rho\frac{d\bar{q}}{dt} = \rho\bar{f} - \nabla p + \kappa(\nabla\times\bar{\upsilon}) + (\mu+\kappa)\nabla\times\nabla\times\bar{q} + (\lambda+2\mu+\kappa)\nabla(\nabla\cdot\bar{q}) + \bar{J}\times\bar{B} \tag{2}$$

$$\rho j\frac{d\bar{q}}{dt} = \rho\bar{l} - 2\kappa\bar{\upsilon} + \kappa(\nabla\times\bar{q}) - \gamma(\nabla\times\nabla\times\bar{\upsilon}) + (\alpha_1+\beta+\gamma)\nabla(\nabla\cdot\bar{\upsilon}) \tag{3}$$

Where ρ and j denote the density and gyration parameters respectively. $\bar{q}$, $\bar{\upsilon}$, $\bar{f}$ and $\bar{l}$ denote the velocity, microrotation, body forces per unit mass and body couple per unit mass respectively. p denotes the fluid pressure at any point. λ, μ and κ denote the coefficients of shear, bulk and vortex viscosities respectively. α_1, β, γ denote the gyro-viscosity coefficients. These coefficients are constrained to,

$$\kappa \geq 0\ ;\ 2\mu + \kappa \geq 0\ ;\ 3\lambda + 2\mu + \kappa \geq 0\ ;\ \gamma \geq 0\ ;\ |\beta\,| \geq \gamma\ ; 3\alpha_1 + \beta + \gamma\ \geq 0 \qquad (4)$$

III. MATHEMATICAL FORMULATION

A laminar, fully developed Poiseuille flow of an incompressible, slightly conducting micropolar fluid flow in a channel bounded below by a porous bed is considered. The interface of the porous bed is coincides with the x-axis and the y-axis is taken normal to the porous bed. The main flow in the channel is driven by a constant pressure gradient which is taken in the x-direction. There is an injection of fluid from the lower porous bed and the suction into the upper plate with the same velocity U in the y-direction. Beavers Joseph condition is used at the fluid-porous bed interface. A magnetic field of strength B_o is applied at an inclined angle θ $(0 \leq \theta < \pi/2)$ with the y-axis. “The induced magnetic effects can be neglected with respect to the applied magnetic effects, as magnetic Reynolds number is smaller than unity” [18].

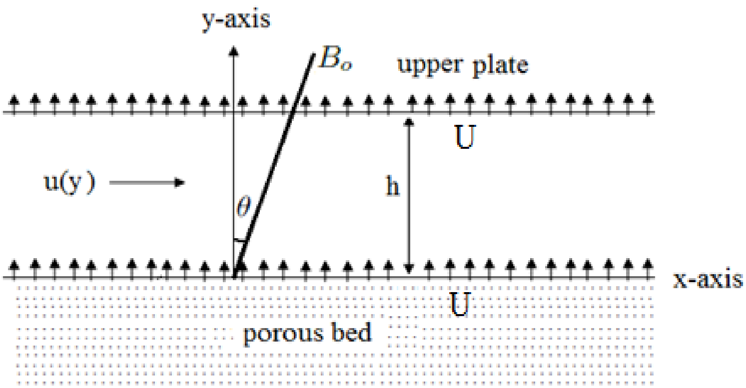

Fig.1. Flow diagram

Owing to the above considerations, we get velocity as $\bar{q} = (u(y)\,, U\,, 0)$ and micro-rotation as $\bar{\upsilon} = (0,\ 0, c\,(y))$. With these, the governing equations of the flow field under consideration in the absence of the body couples are

$$\rho U \frac{du}{dy} = -\frac{dp}{dx} + \kappa \frac{dc}{dy} + (\mu + \kappa)\frac{\mathrm{d}^2\mathrm{u}}{\mathrm{dy}^2} - \sigma_e (B_0 \cos\theta)^2 u \qquad (5)$$

$$\rho j U \frac{dc}{dy} = -2\kappa c - \kappa \frac{du}{dy} + \gamma \frac{\mathrm{d}^2\mathrm{c}}{\mathrm{dy}^2} \qquad (6)$$

and the boundary conditions are

$$\frac{du}{dy} = \frac{\alpha}{\sqrt{\kappa}}(u_s - D) \quad \text{at} \quad y = 0 \qquad \text{(BJ slip condition)} \qquad (7)$$

$$u(h) = 0 \qquad \text{(No slip condition)} \qquad (8)$$

$$c(0) = 0 = c(h) \qquad \text{(Vanishing of microrotation)} \qquad (9)$$

where, $u_s = u_{at\ y=0}$ (slip velocity), $D = -\frac{\kappa}{\mu}\frac{dp}{dx}$ (Darcy’s velocity), σ_e is the electrical conductivity of the micropolar fluid, α is slip coefficient and k is permeability of the porous bed.

Introducing the non-dimensional quantities: $hx^* = x,\ hy^* = y,\ Uu^* = u\ ,\ Uc^* = hc,\ \rho U^2 p^* = p$, the governing equations of flow with the boundary conditions (after dropping *’s) are given by

$$\frac{\mathrm{d}^2\mathrm{u}}{\mathrm{dy}^2} + m\frac{dc}{dy} - R\frac{du}{dy} - M^2 u = RP \qquad (10)$$

$$\frac{\mathrm{d}^2\mathrm{c}}{\mathrm{dy}^2} - n\frac{du}{dy} - RJ\frac{dc}{dy} - 2nc = 0 \qquad (11)$$

$$\frac{du}{dy} = \alpha\sigma\left(u_s - \frac{RP}{\sigma^2}\right) \quad \text{at } y = 0 \qquad (12)$$

$$u(1) = 0 \qquad (13)$$

$$c(0) = 0 = c(1) \tag{14}$$

where: Reynolds number $R = \frac{\rho U h}{(\mu+\kappa)}$, coupling parameter $m = \frac{\kappa}{(\mu+\kappa)}$, gyration-parameter $n = \frac{h^2\kappa}{\gamma}$, microrotation-parameter $J = \frac{j(\mu+\kappa)}{\gamma}$, Hartmann number $Ha = B_0 h\sqrt{\frac{\sigma_e}{\mu}}$, porosity parameter $\sigma = \frac{h}{\sqrt{\kappa}}$, pressure gradient $P = \frac{dp}{dx}$ and $M = Ha\, cos\theta$.

IV. SOLUTION OF THE PROBLEM

From Eq.(10) and Eq.(11), we observe that the velocity u(y) satisfies the following fourth order ODE

$$u^{iv} - Au''' + Bu'' + Cu' + Du = 2nRP\ , \tag{15}$$

where $A = R(J+1)\ , B = ((m-2)n - M^2 + R^2J)\ , C = R(2n + M^2J)\ , D = 2M^2n\ ,$
and its general solution can be taken as

$$u(y) = C_1e^{m_1y} + C_2e^{m_2y} + C_3e^{m_3y} + C_4e^{m_4y} + \frac{2nRP}{D} \tag{16}$$

Now, from Eq.(10) and Eq.(11) , we have c(y) as

$$c(y) = \frac{1}{2mn}[Au'' - u''' - (B+2n)u' - R(JM^2 - P)]\ , \tag{17}$$

and substituting the expressions of u', u'' and u''' in c(y), we have

$$c(y) = F_1e^{m_1y} + F_2e^{m_2y} + F_3e^{m_3y} + F_4e^{m_4y} \tag{18}$$

with $F_j = \frac{C_j}{2nm}\left((A - m_j)m_j^2 - (2n+B)m_j - RJM^2\right), j = 1,2,3,4$

$m_{1,2} = (\frac{A}{2} - A_4 \pm \sqrt{A_5 - A_6})/2,\ m_{3,4} = (\frac{A}{2} + A_4 \pm \sqrt{A_5 + A_6})/2,$

$A_1 = (B^2 + 3AC + 12D),\ A_2 = (2B^3 + 9ABC + 27C^2 + 27A^2D - 72BD),$

$A_3 = \sqrt[3]{(A_2 + \sqrt{A_2^2 - 4A_1^3})/2}\ ,\ A_4 = \sqrt{\frac{A_2}{4} - (2B + A_2 + A_1/A_3)/3}\ ,$

$A_5 = (3A^2/4 - 2B - A_4^2),\ A_6 = (A^3 - 4AB - 8C)/(4A_4),$

and $C_j, j = 1,2,3,4$ are not given here for brevity.

V. DISCUSSION OF RESULTS

The Poiseuille flow of micropolar fluid in a horizontal channel with lower porous bed under the effect of inclined magnetic field is considered. The resulting flow equations are analytically solved and closed form expressions for the velocity and micro-rotation are obtained. The effects of "pertinent parameters" entering into the problem on the velocity $u(y)$ and microrotation $c(y)$ are studied and presented through Figs. 2 - 8. For the result analysis purpose, we consider that the parameters have fixed values as $\alpha = 0.5$, $R = 1$, $Ha = 2$, $P = 1$, $\theta = \frac{\pi}{4}$, $\sigma = 5$,$J = 1$, $M = 2$, $m = 0.5$, $n = 0.5$. Further, if any variation in the parameters is indicated in that particular figure.

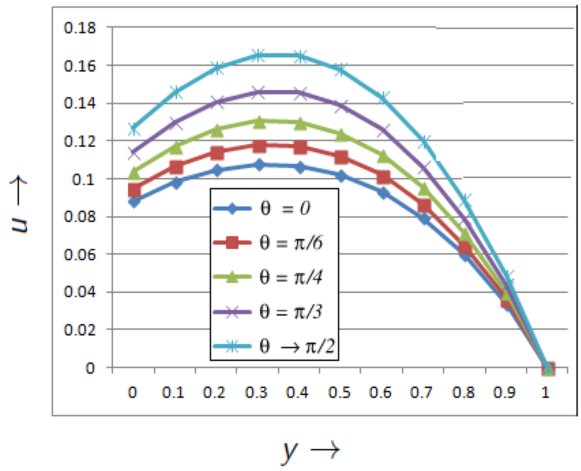

Fig.2. Effects of θ on $u(y)$

Fig. 2 depicts the effect of the inclination angle θ ($0 \le \theta < \pi/2$) on velocity. The velocity is increasing, as θ is increasing. When $\theta = 0$, the flow is under the influence of the transverse magnetic field. As $\theta \to \frac{\pi}{2}$, (i.e., $Ha\cos\theta \to 0$) , the flow is with no magnetic field. From Fig. 3, we see that the velocity is decreasing, as magnetic parameter M is increasing. In the fluid flow, it signifies the retarding effect of the magnetic field.

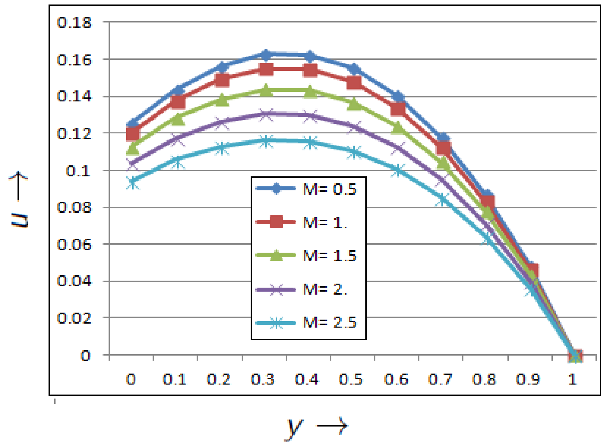

Fig.3. Effects of M on $u(y)$

In Fig. 4 we see that as J is increasing, the velocity is also increasing and this increase is more profound in the lower half of the channel near the porous bed. From Fig.5, we see that the velocity is decreasing, as σ is increasing. As $\sigma \to \infty$, it leads to the fluid flow in a parallel plate channel.

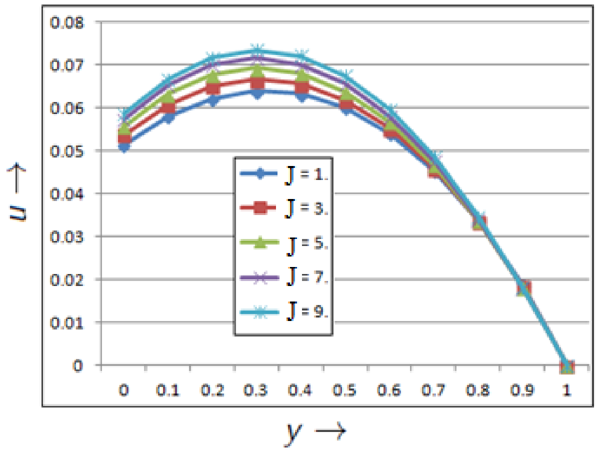

Fig.4. Effects of J on $u(y)$

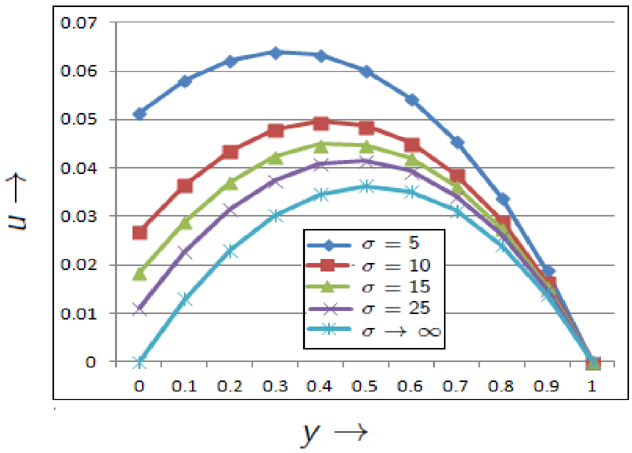

Fig.5. Effects of σ on $u(y)$

From Fig. 6, we see that the microrotation is increasing, as the inclination angle θ $(0 \le \theta < \pi/2)$ is increasing. The increase in microrotation is more near the upper plate in comparison with the lower porous bed. In Fig. 7, we see that the microrotation is decreasing, as magnetic parameter M is increasing. In Fig.8, we see that as J is increasing, microrotation is increasing and the increase is more profound near the rigid plate. From Fig.9, we see that the microrotation is increasing, as σ is increasing.

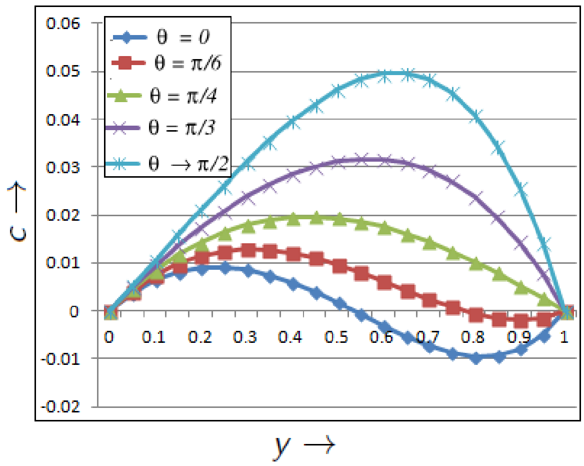

Fig.6. Effects of θ on $c(y)$

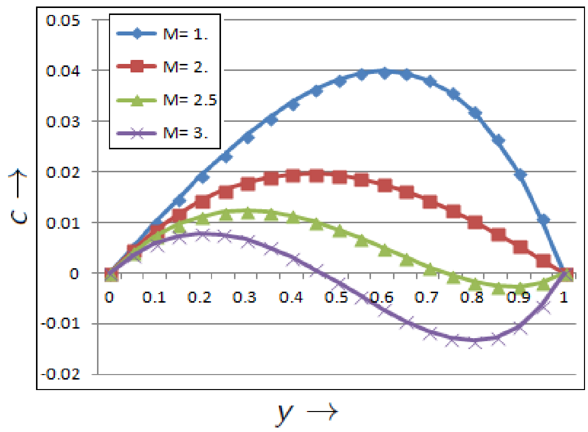

Fig.7. Effects of M on $c(y)$

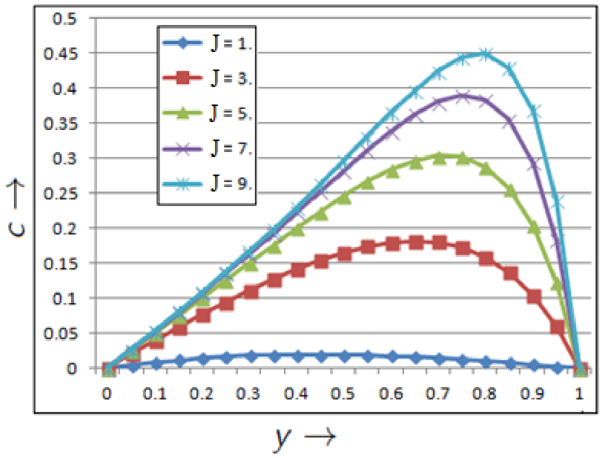

Fig.8. Effects of J on $c(y)$

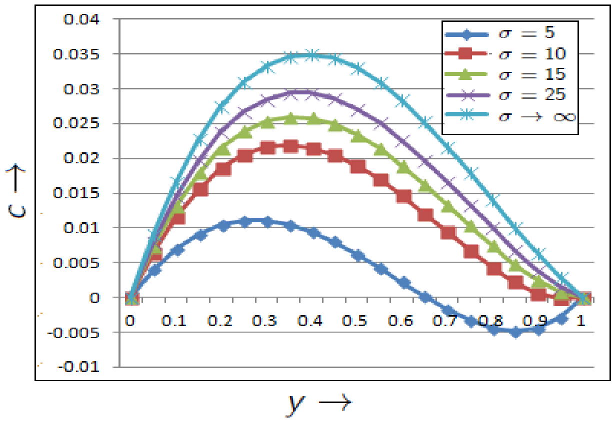

Fig.9. Effects of σ on $c(y)$

VI. CONCLUSION

Inclined magnetic effects on micropolar fluid flow in a channel bounded below by a porous bed are studied. The observations of the current study are:

1. The velocity is increased by the increase of angle of inclination,θ $(0 \leq \theta < \pi/2)$ and micro-polarity parameter where as it is decreased by the increase of magnetic and porosity parameters.
2. As $\sigma \to \infty,$ the fluid velocity corresponds to the flow in a plane channel.
3. The microrotation is increased by the increase of angle of inclination, micro-polarity parameter, and porosity parameter where as it is decreased by the increase of magnetic parameter.

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